Senior 4 Pre-Calculus Mathematics (40S) Standards Test

Student Booklet (Part 1)

January 2006

Manitoba

Building for the Future

Senior 4 Pre-Calculus Mathematics (40S)
Standards Test

DESCRIPTION

Total Possible Marks: 100 **Time: 3 hours**

Part	Description	Time	Value
Part 1	• **Long Answer:** 11 questions worth a total of 35 marks Calculators (scientific or graphing) are permitted.	60 minutes	35%
Part 2	• **Multiple Choice:** 15 questions worth 1 mark each • **Short Answer:** 15 questions worth 1 mark each • **Long Answer:** 10 questions worth a total of 35 marks Calculators are not permitted.	120 minutes	65%

GENERAL DIRECTIONS

- Formulas are given on the tear-out sheet at the beginning of this test booklet. Please do not tear out any pages other than this formula sheet.
- Read all instructions carefully.
- The blank pages at the back of each booklet may be used for rough work. No marks will be given for work done on these pages.
- Note that diagrams and graphs provided in the test booklets may not be drawn to scale.
- When the *Student Booklet (Part 2)* is distributed, you may keep the *Student Booklet (Part 1)* in case you wish to continue working on it without a calculator.

106-03894/

Formula Sheet—Pre-Calculus Mathematics (40S)

$$s = \theta r$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$\cos 2\alpha = 1 - 2\sin^2\alpha$$

$$\cos 2\alpha = 2\cos^2\alpha - 1$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

$$P(n,r) \text{ or } {}_nP_r = \frac{n!}{(n-r)!}$$

$$C(n,r) \text{ or } {}_nC_r = \frac{n!}{r!(n-r)!}$$

$$t_{k+1} = {}_nC_k a^{n-k} b^k$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ and } B) = P(A) \bullet P(B|A)$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = Pe^{rt}$$

$$e = 2.718\ 28$$

$$\log_a(MN) = \log_a M + \log_a N$$

$$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$$

$$\log_a(M^n) = n\log_a M$$

$$\log_a M = \frac{\log_b M}{\log_b a}$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1, \quad a > b$$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1, \quad a > b$$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$(y-k) = a(x-h)^2$$

$$(x-h) = a(y-k)^2$$

$$t_n = t_1 r^{n-1}$$

$$S_n = \frac{t_1(1-r^n)}{1-r} = \frac{t_1(r^n-1)}{r-1}$$

$$S_n = \frac{t_1 - t_n r}{1-r} = \frac{t_n r - t_1}{r-1}$$

$$S_\infty = \frac{t_1}{1-r}, \quad |r| < 1$$

Long-Answer Questions

Instructions

- There are 11 long-answer questions for a total of 35 marks.
- Calculators (scientific or graphing) are allowed for this part of the test.
- Write each solution in the space provided.
- For full marks, your answers must show all pertinent diagrams, calculations, and explanations.
- Graphing calculator solutions should include an explanation of how your final answer is obtained.
- Your solutions should be neat, organized, and clear.
- If any curve contains asymptotes, the asymptotes should be included in the graph.

DO NOT USE

Long-Answer Questions

1. Solve the logarithmic equation: 101

$$\log_3(x+2)+\log_3(2x+1)-2=0$$

(4 marks)

DO NOT USE

102

2. A student randomly selects one of the following three boxes.
The boxes contain the number of coloured marbles as indicated:

Box A	Box B	Box C
3 red 2 blue	4 red 5 blue	3 red 3 blue

Given that a red marble has been selected, find the probability that it was chosen from Box B.

Express your answer as an exact value or correct to 3 decimal places.

(4 marks)

DO NOT USE

3. Brandy, Leo and 6 of their friends attend a movie. 103

In how many ways can they be seated in a row so that Brandy and Leo do not sit next to each other?

(2 marks)

4. Solve the following equation algebraically:

$$3 \cdot 9^{x} = 5^{(2-x)}$$

Express your answer correct to 3 decimal places.

(4 marks)

DO N
USI

104

DO NOT USE

105

5. Solve the following equation:

$$\tan^2 \theta + 3 = 7$$

Give the general solution in radian measure correct to 3 decimal places.

(4 marks)

DO NOT USE

6. The 6^{th} term of the binomial expansion of $\left(x^3 - \frac{4}{x}\right)^n$ contains x^{16}. 106

Find the value of n.

(3 marks)

7. An initial population of a city was 5 000 and grew exponentially to 23 000 in 9 years. The population growth can be represented by the equation $A = Pe^{rt}$ where r is the rate of growth and t represents the number of years. 107

Find the rate of growth for this city correct to 3 decimal places.

(3 marks)

DO NOT USE

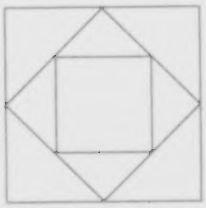
8. Box A contains 4 red and 3 green marbles and Box B contains 2 red and 2 green marbles. Two marbles are selected, one from each box. 108

Find the probability that the marbles are of different colours.

(3 marks)

9. A square has a side length of 2 units. A new square is formed by joining the midpoints of each side of the original square. This process is continued infinitely. 109

Find the sum of the perimeters of all the squares.



(3 marks)

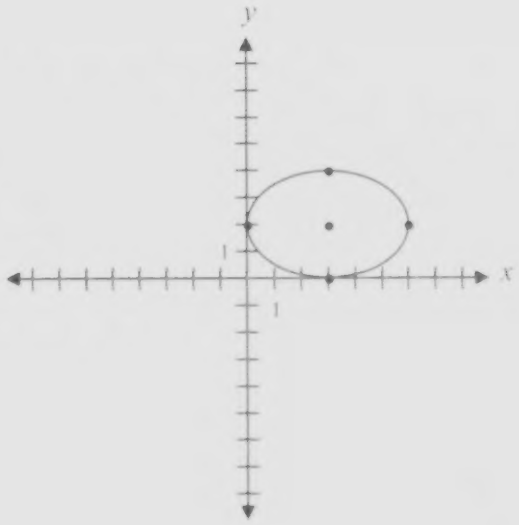
DO NOT USE

10. There are 35 students in a room; 18 of the students are girls and 17 are boys. Three students are chosen at random for a committee. 110

What is the probability that exactly 2 of the students are girls?
Briefly explain your calculations.

(3 marks)

11. Find the equation, in standard form, of the conic section sketched below: 111



(2 marks)

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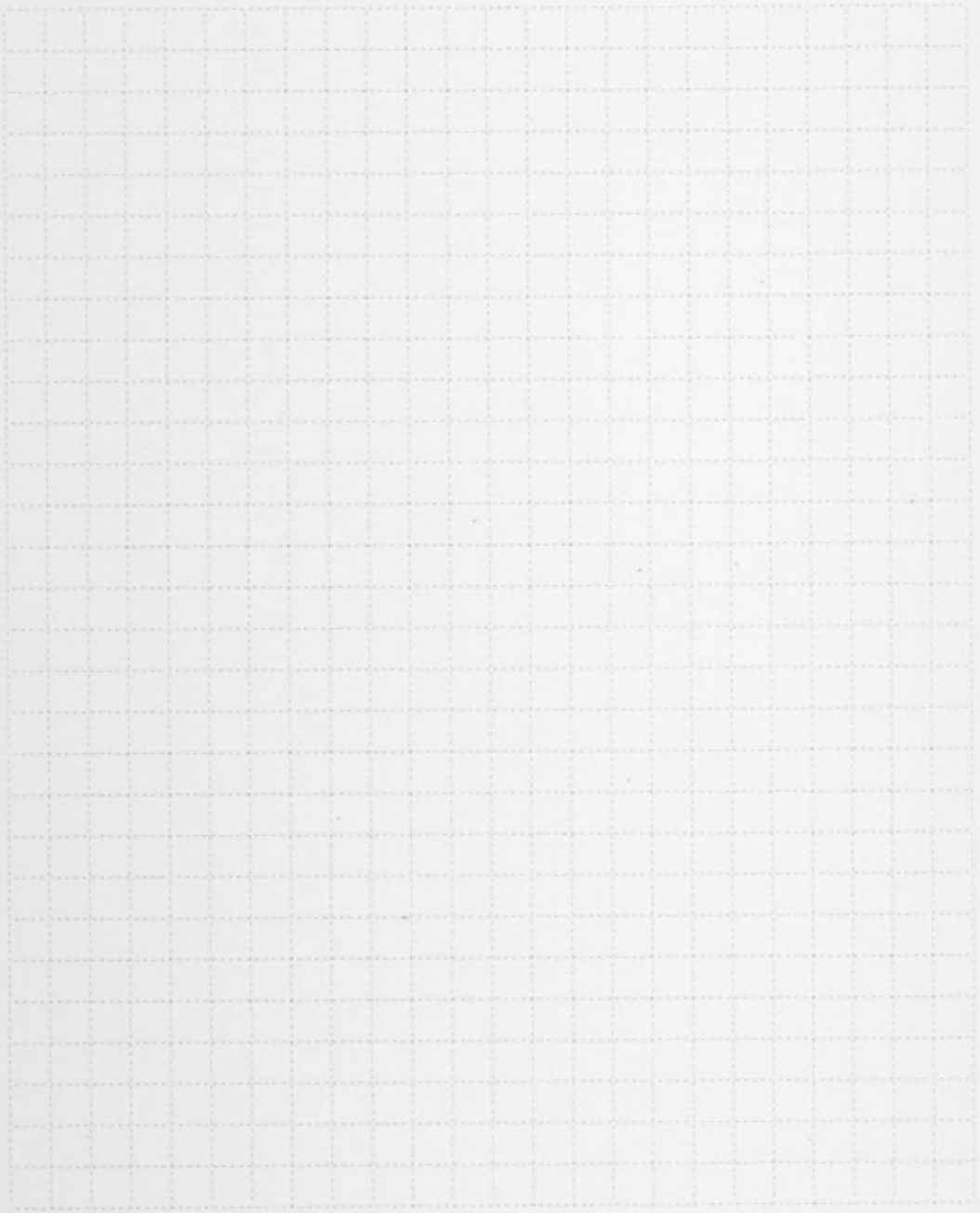

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Senior 4 Pre-Calculus Mathematics (40S) Standards Test

Student Booklet (Part 2)

January 2006

Manitoba
Building for the Future

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Multiple-Choice Questions

Instructions

- There are 15 multiple-choice questions.
- Each question is worth 1 mark.
- Read each question carefully.
- Examine all the choices before you decide on your answer.
- You may use the spaces beside each question for rough work.
- Record your answers on the sheet provided.
- Provide only one answer per question.
- There is no penalty for guessing.
- Answer all questions.
- Calculators (scientific or graphing) are **not** allowed for this part of the test.

Multiple-Choice Questions

12. If $P(\theta)$ is a point on the unit circle, then the coordinates of $P\left(\frac{5\pi}{6}\right)$ are:

a) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

b) $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

c) $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

d) $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

13. The solution to the trigonometric equation $\sin 2\theta = 1$ in the interval $[0°, 360°]$ is:

a) $\{30°, 150°\}$

b) $\{45°, 225°\}$

c) $\{60°, 120°\}$

d) $\{90°, 450°\}$

14. If $\sin\theta > 0$ and $\sec\theta > 0$, then θ terminates in quadrant:

a) I

b) II

c) III

d) IV

15. The period and phase shift for the trigonometric equation $y = 2\sin 2\left(\theta - \frac{\pi}{2}\right)$ are:

a) period $= \pi$; phase shift $= \frac{\pi}{2}$ left

b) period $= \pi$; phase shift $= \frac{\pi}{2}$ right

c) period $= 2\pi$; phase shift $= \frac{\pi}{2}$ left

d) period $= 2\pi$; phase shift $= \frac{\pi}{2}$ right

16. One ordered pair for a function f is $(2, -5)$. Given that f is an odd function, which of the following must be another ordered pair for the function f?

a) $(-5, 2)$

b) $(2, 5)$

c) $(-2, -5)$

d) $(-2, 5)$

17. The range of the function $y = |x + 2| - 1$ is:

a) $[-1, \infty)$

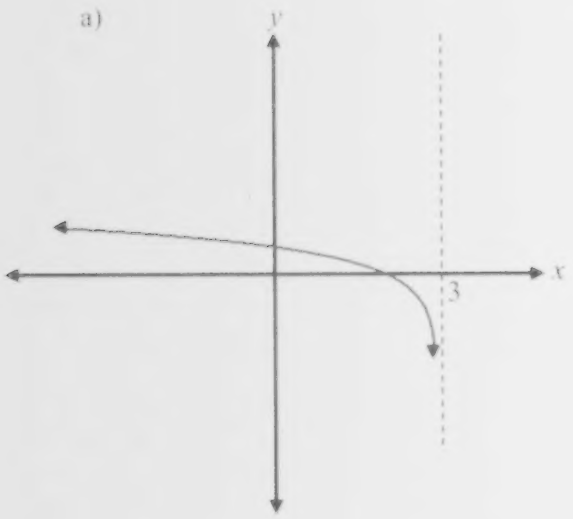
b) $[1, \infty)$

c) $[-2, \infty)$

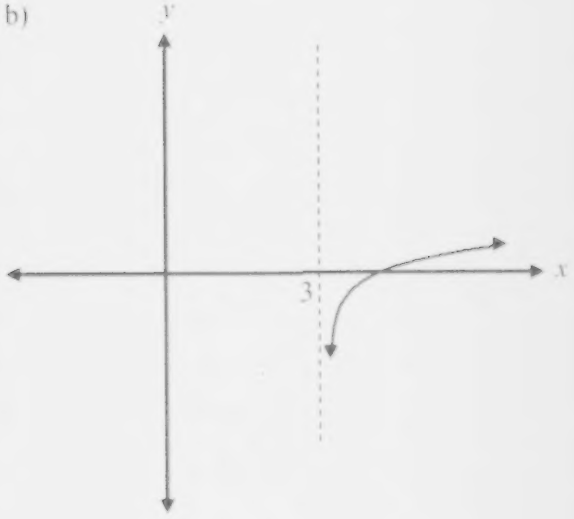
d) $[2, \infty)$

18. The graph of the function $f(x)=\log_4\left[-(x-3)\right]$ is best described by which of the following?

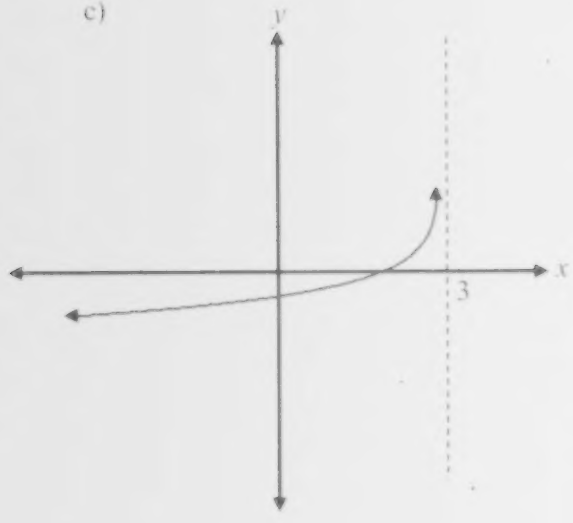
a)



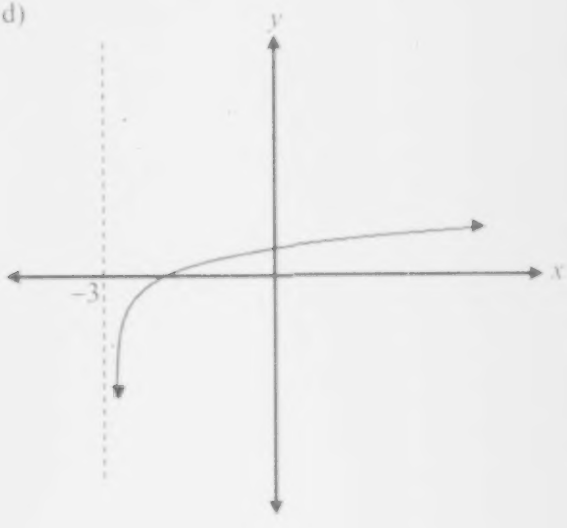
b)



c)



d)



19. If $\log_m 2 = r$ and $\log_m 5 = s$, then $\log_m 50$ is:

a) $2s + r$

b) $s^2 + r$

c) $s^2 r$

d) $\dfrac{s^2}{r}$

20. If all the following letters S L E U F I D F are used, how many 8 letter arrangements are possible?

a) $\frac{7!}{2!}$

b) $7!2!$

c) $8!2!$

d) $\frac{8!}{2!}$

21. In the binomial expansion of $(3x+2)^8$, the last term is:

a) $3 \cdot 2^8$

b) 2^8

c) 2

d) 2^0

22. If there are 3 identical red books and 4 identical green books on a shelf, how many arrangements are possible keeping all the red books together?

a) $5!$

b) $\frac{7!}{3!4!}$

c) $\frac{5!}{4!}$

d) $3!4!$

23. The conic section $4x^2 - 3y^2 - 4x + 6y - 10 = 0$ is a:

a) parabola

b) circle

c) ellipse

d) hyperbola

24. If one number is to be selected from each list below, what is the probability that each number selected will be a 5 or more?

List I	List II
2, 4, 6	3, 5, 6

a) $\frac{1}{9}$

b) $\frac{2}{9}$

c) $\frac{3}{9}$

d) $\frac{7}{9}$

25. If $P(A)=\frac{3}{4}$ and $P(A \text{ and } B)=\frac{1}{2}$, where A and B are dependent events, then $P(B|A)$ is:

a) $\frac{1}{4}$

b) $\frac{3}{8}$

c) $\frac{2}{3}$

d) $\frac{4}{5}$

26. The value of $\sum_{k=0}^{3}\left(2^{k}\right)$ is:

a) 8

b) 14

c) 15

d) 64

Short-Answer Questions

Instructions

- There are 15 short-answer questions.
- Each question is worth 1 mark.
- Calculators are **not** allowed for this part of the test.
- Write your final answer in the space provided.
- Work need not be shown in order to obtain full marks for a correct answer. However, in the case of an incorrect final answer, part marks may be awarded only if work is shown.
- Solutions need not have rationalized denominators.

DO NOT USE

Short-Answer Questions

27. Evaluate: $\ln e^2$ 112

(1 mark)

28. If the coordinates of a point $P(\theta)$ on the unit circle are $(a, -b)$, what will be the coordinates of the point $P(\theta - \pi)$? 113

(1 mark)

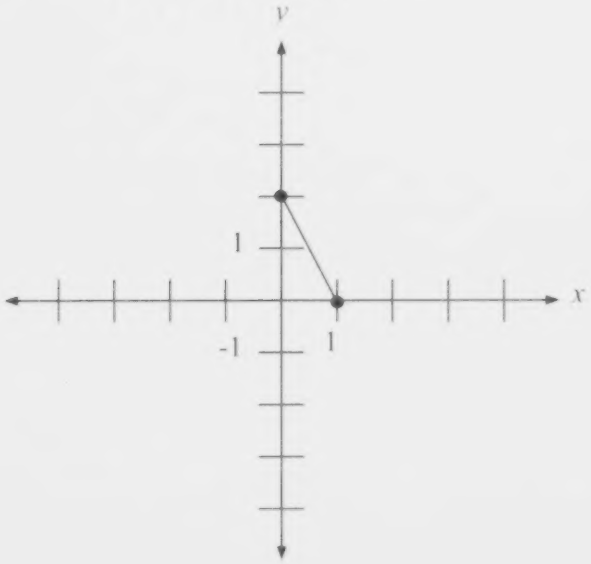
29. Give the period of the function: 114

$$y = \sec\theta$$

(1 mark)

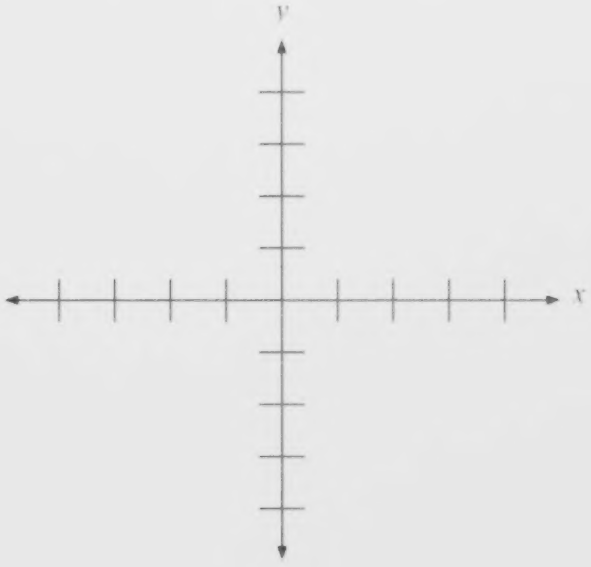
DO NOT USE

30. Given the graph of $y = f(x)$ below:



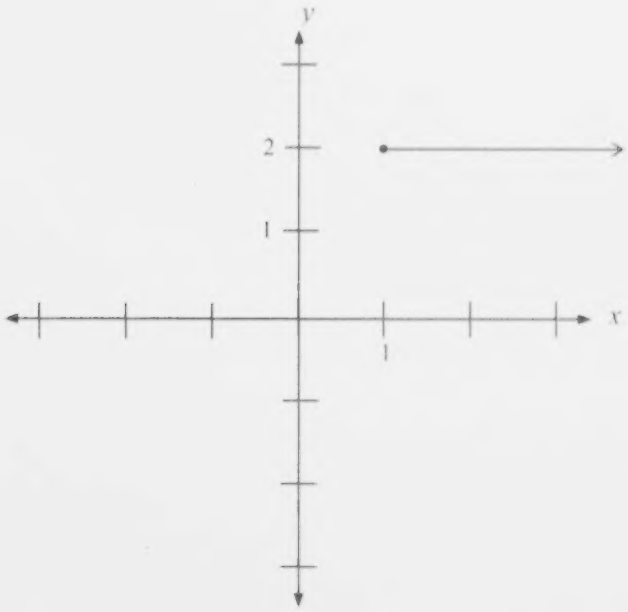
Sketch the graph of $y = f^{-1}(x)$. 115

(1 mark)



DO NOT USE

31. Given the graph of $y = f(x)$ below:



Sketch the graph of $y = \frac{1}{f(x)}$.

(1 mark)

116

DO NOT USE

32. Express $\cot^2\theta$ in terms of $\cos\theta$ only. 117

(1 mark)

33. Convert $\frac{4\pi}{5}$ radians to degree measure. 118

Express your answer as a whole number.

(1 mark)

34. Solve for x in the equation: 119

$$3^x = \frac{1}{27}$$

(1 mark)

DO NOT USE

35. Solve for n in the equation: 120

$$\frac{(n-1)!}{n!} = \frac{1}{5}$$

(1 mark)

36. In how many ways can 5 people be seated at a round table? 121

(1 mark)

37. How many terms are in the binomial expansion of $\left(2x^2 + \frac{2}{x}\right)^6$? 122

(1 mark)

DO NOT USE

38. Give the coordinates of the centre for the conic section described by the equation: 123

$$\frac{(x-1)^2}{9} + \frac{y^2}{16} = 1$$

(1 mark)

39. State the equation of an axis of symmetry for the conic section described by the equation: 124

$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$

(1 mark)

DO NOT USE

40. The probability that Leigh will be on time is 0.8 and the probability that she will wear blue is 0.3. 125

What is the probability that she will be late and wear blue?

(1 mark)

41. The first 3 terms of a geometric sequence are $\log 2$, $\log 2^{\frac{2}{3}}$, $\log 2^{x}$. 126

What is the value of x in this sequence?

(1 mark)

Long-Answer Questions

Instructions

- There are 10 long-answer questions worth a total of 35 marks.
- Calculators are **not** allowed for this part of the test.
- Write each solution in the space provided.
- For full marks, your answers must show all pertinent diagrams, calculations, and explanations.
- Your solutions should be neat, clear and well organized.
- If any curve contains asymptotes, the asymptotes should be included in the graph.
- Solutions need not have rationalized denominators.

DO NOT USE

Long-Answer Questions

42. Prove the identity:

$$\frac{\sin x}{\cos x + 1} + \frac{\cos x + 1}{\sin x} = 2\csc x$$

(4 marks)

127

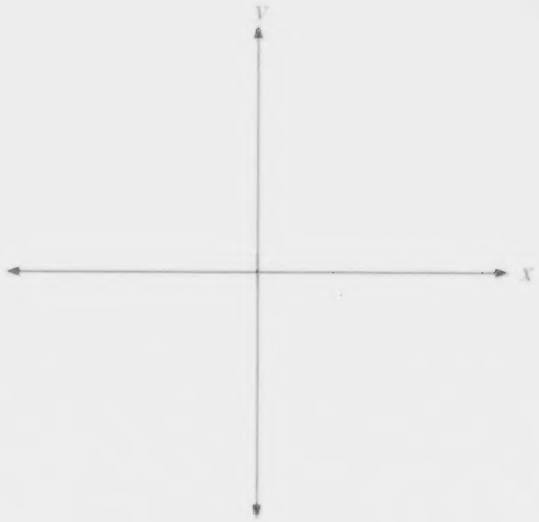
DO NOT USE

43. a) State the domain of $y = \tan x$. 128

(1 mark)

b) Sketch at least one period of the graph of $y = \tan x$. 129

(1 mark)



c) Sketch the graph of $y = \tan^{-1} x$. 130

(1 mark)

DO NOT USE

44. a) Find the exact value of:

131

$$\cos\left(\frac{11\pi}{12}\right)$$

(3 marks)

b) Find the exact value of:

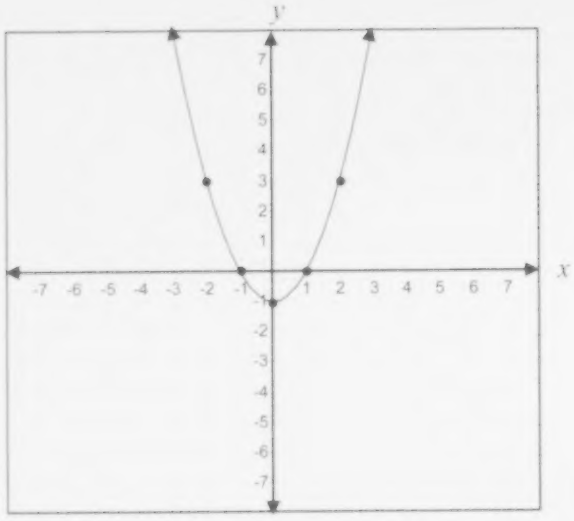
132

$$\sec\left(\frac{11\pi}{12}\right)$$

(1 mark)

DO NOT USE

45. Given the graph of $y = f(x)$ below:

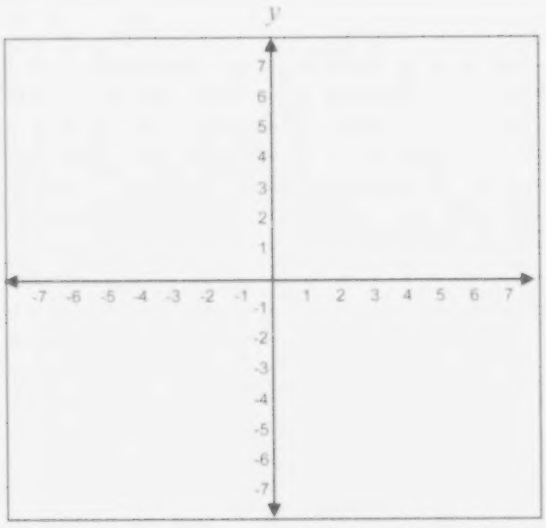


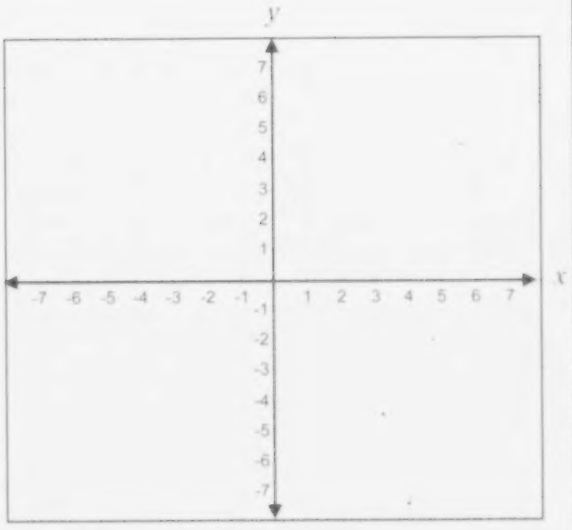
Sketch the graphs of :

a) $y = 2f(x+1)$ (2 marks)

b) $y = -|f(x)|$ (2 marks)

133
134





DO NOT USE

135

46. Solve for θ in the interval $[0, 2\pi]$:

$$2\cos^2\theta + 3\cos\theta + 1 = 0$$

(3 marks)

DO NOT USE

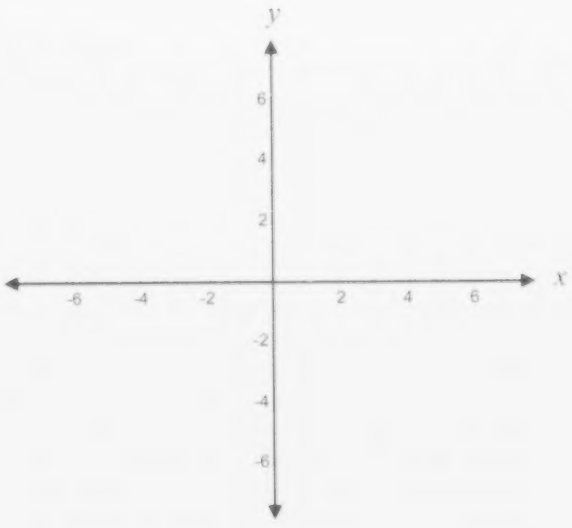
47. A conic section is represented by the equation $4x^2 - 8x - y^2 + 2y - 13 = 0$.

a) Write the equation in standard form. 136

(3 marks)

b) Sketch the graph of this conic section. 137

(2 marks)



DO NOT USE

138

48. Express $\frac{1}{2}\log A - 3\log B$ as a single logarithm.

(2 marks)

DO NOT USE

49. If $\sin\theta = \frac{4}{5}$ and $\sec\theta < 0$ find the exact value of:

a) $\tan 2\theta$ 139

(3 marks)

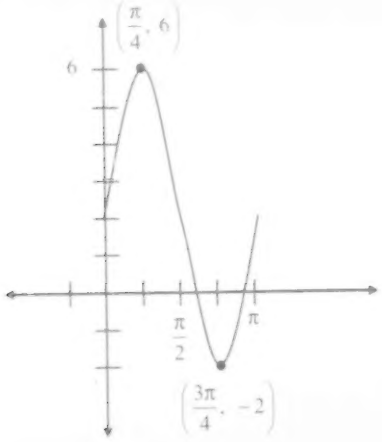
b) $\sin 2\theta$ 140

(1 mark)

DO NOT USE

50. The trigonometric graph shown has one maximum value at the point $\left(\frac{\pi}{4}, 6\right)$ and the next minimum value at the point $\left(\frac{3\pi}{4}, -2\right)$. 141

Find an equation for this graph in terms of the cosine function.



(3 marks)

DO NOT USE

51. Given the function $y = \sqrt{x+1} - 2$:

a) the x-intercept is _____ 142

(1 mark)

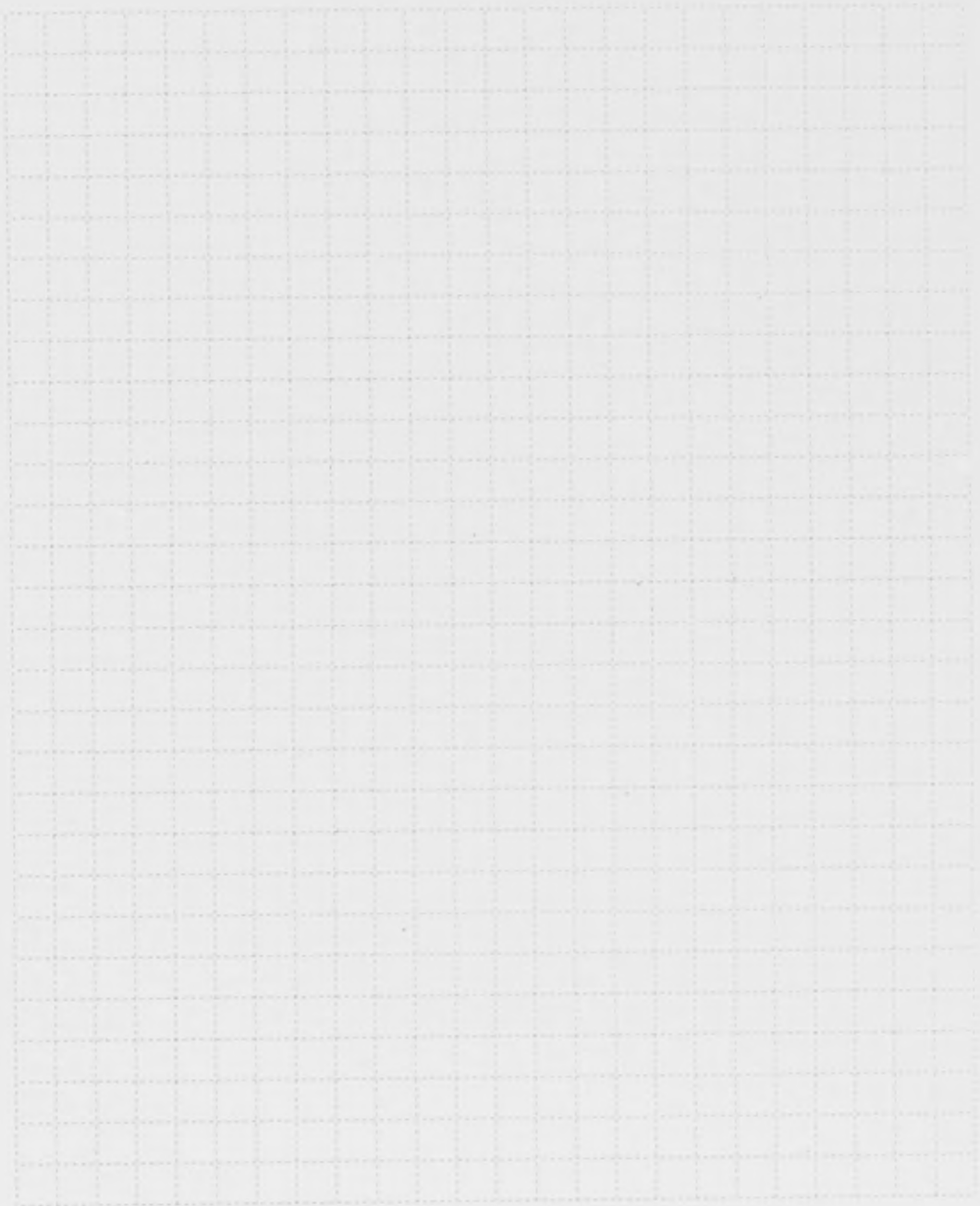
b) the y-intercept is _____ 143

(1 mark)

c) Sketch a clearly labelled graph of the function. 144

(1 mark)

NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE.



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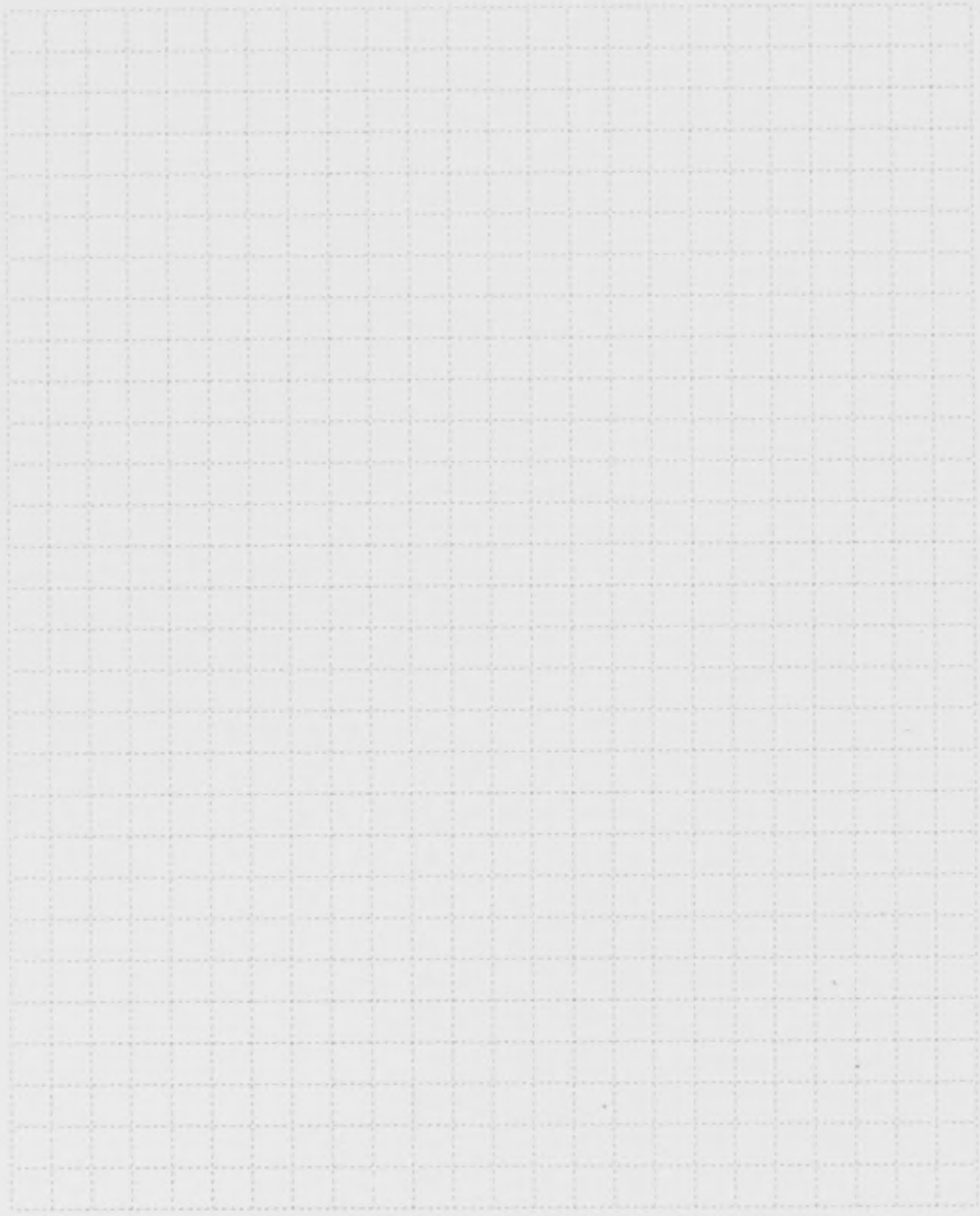

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Marking Guide

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General Marking Instructions

Please make no marks in the student test booklets. If the booklets have marks in them, the marks need to be removed by departmental staff prior to sample marking should the booklet be selected.

Please ensure that:

- the student test booklet number and the number on the *Answer/Scoring Sheet* are identical;
- **only a pencil is used to complete the *Answer/Scoring Sheets*, by both students and markers;**
- the totals of each of the four parts are written at the bottom;
- each student's final result is recorded, by test booklet number, on the corresponding ***Answer/Scoring Sheet***;
- the grand total is written in the appropriate location;
- the *Answer/Scoring Sheet* is complete; and
- a photocopy has been made for school records.

Once marking is completed, please forward the ***Answer/Scoring Sheets*** to Manitoba Education, Citizenship and Youth in the envelope provided (for more information see the *Administration Manual for Supervising Teachers*).

Scoring the Multiple-Choice Questions

Please follow the instructions on the ***Answer Key for Multiple-Choice Questions*** on page 5.

Scoring the Short-Answer and Long-Answer Questions

The remainder of the Senior 4 Pre-Calculus Mathematics (40S) Standards Test is composed of short-answer questions and long-answer questions. Short-answer questions are worth 1 mark each and long-answer questions are worth 2 to 5 marks each.

Each question is designed to elicit a well-defined response according to the associated specific learning outcome(s) and relevant mathematical processes. Their purpose is to determine whether a student meets the standards for the course as they relate to the knowledge and skills associated with the question.

To receive full marks, a student's response must be complete and correct. Where alternative answering methods are possible, the *Marking Key* attempts to address the most common solutions. For general guidelines regarding the scoring of students' responses, see Appendix A.

Irregularities in Standards Tests

During the administration of standards tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix B provides examples of such irregularities as well as procedures to follow to report irregularities.

If an *Answer/Scoring Sheet* is marked with "NR" and/or "0" only (e.g., student was present but did not attempt any questions) please document this on the *Irregular Test Paper Report*.

Assistance

If, during marking, any marking issue arises that cannot be resolved locally, please call Manitoba Education, Citizenship and Youth at the earliest opportunity to advise and/or seek assistance. If required, we will connect you to an advisor. Also, please be sure to notify us before making any modifications to the answer keys or scoring rubrics. Contact person: Chantal Bérard at 945-7590 or toll-free at 1 800 282-8069, extension 7590.

SCORING GUIDELINES

Answer Key for Multiple-Choice Questions

Answer Key for Multiple-Choice Questions

If you are using this sheet by punching out holes corresponding to the correct answers and overlaying this sheet on the *Answer/Scoring Sheet*, then

1) check the student's responses for multiple bubbles first (questions with multiple bubbles are to be scored as "0");

2) overlay this page on the *Answer/Scoring Sheet*; and

3) count the number of correct responses, excluding questions with multiple responses, if any.

You may also score the multiple-choice questions by making a transparency of this page and overlaying it on the *Answer/Scoring Sheet*.

Remember to write the total score for the multiple-choice questions at the bottom of the *Answer/Scoring Sheet*.

MULTIPLE-CHOICE QUESTIONS / QUESTIONS À CHOIX MULTIPLE

Fill in the best answer for each question. / Choisir la meilleure réponse pour chaque question.

12	Ⓐ**Ⓑ**ⒸⒹ	15	Ⓐ**Ⓑ**ⒸⒹ	18	**Ⓐ**ⒷⒸⒹ	21	Ⓐ**Ⓑ**ⒸⒹ	24	Ⓐ**Ⓑ**ⒸⒹ
13	Ⓐ**Ⓑ**ⒸⒹ	16	ⒶⒷⒸ**Ⓓ**	19	**Ⓐ**ⒷⒸⒹ	22	Ⓐ Ⓑ **Ⓒ** Ⓓ	25	Ⓐ Ⓑ **Ⓒ** Ⓓ
14	**Ⓐ**ⒷⒸⒹ	17	**Ⓐ**ⒷⒸⒹ	20	ⒶⒷⒸ**Ⓓ**	23	Ⓐ Ⓑ Ⓒ **Ⓓ**	26	Ⓐ Ⓑ **Ⓒ** Ⓓ

Multiple-Choice Questions			
Question	***Outcome***	***Question***	***Outcome***
12	A3	20	E1
13	A4	21	E4
14	A2	22	E2
15	B7	23	F1
16	B3	24	G2
17	B6	25	G4
18	D4	26	H2
19	D5		

SCORING GUIDELINES

Marking Key
including student exemplars

Part 1—Long-Answer Questions

Question No. 1

Outcome: D3, D6

Solve the logarithmic equation:

$$\log_3(x+2)+\log_3(2x+1)-2=0$$

Solution

Method 1

$\log_3[(x+2)(2x+1)]=2$ — 1 mark for log theorem

$3^2=(x+2)(2x+1)$ — 1 mark for changing to exponential

$9=2x^2+x+4x+2$

$0=2x^2+5x-7$

$0=(2x+7)(x-1)$

$x=-\frac{7}{2}$ $x=1$ — 1 mark for solving for x

1 mark for discarding an extraneous root

4 marks

Method 2–Graphing Calculator

$\log_3[(x+2)(2x+1)]=2$ — 1 mark for log theorem

$3^2=(x+2)(2x+1)$ — 1 mark for exponential form

graph $y=9$ and $y=(x+2)(2x+1)$

intersections occur at $(-3.5, 9)$ and $(1, 9)$ — 1 mark for solving for x

$\log_3(-3.5+2)$ is not defined

the solution is $x=1$ — 1 mark for discarding extraneous root

4 marks

Note(s):

- An equivalent solution would be graphing $y=2x^2+5x-7$ and finding zeroes. There would still be one mark for solving for the two values of x.

EXEMPLAR

$$\log_3 \frac{(x+2)(2x+1)}{} = 2$$

$$3^2 = (x+2)(2x+1)$$

$$9 = 2x^2 + x + 4x + 2$$

$$2x^2 + 5x - 7 = 0$$

$$(2x^2 - 7x)(+2x - 7) = 0$$

$$x(2x-7) + 1(2x-7) = 0$$

$$(x+1)(2x-7) = 0$$

~~$x = -1$~~ $\quad \boxed{x = \frac{7}{2}}$

Mark: **3 out of 4**

Note(s):

- deducted ½ mark for sign error on line 5
- deducted ½ mark for incorrect notation on line 5

Question No. 2

Outcome: G4

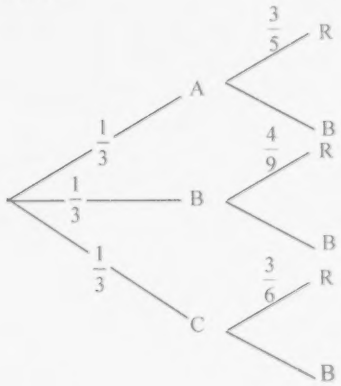
A student randomly selects one of the following three boxes.
The boxes contain the number of coloured marbles as indicated:

Box A	Box B	Box C
3 red 2 blue	4 red 5 blue	3 red 3 blue

Given that a red marble has been selected, find the probability that it was chosen from Box B.

Express your answer as an exact value or correct to 3 decimal places.

Solution



Method 1

$$P(\text{box B}|R)=\frac{\left(\frac{1}{3}\right)\left(\frac{4}{9}\right)}{\left(\frac{1}{3}\right)\left(\frac{3}{5}\right)+\left(\frac{1}{3}\right)\left(\frac{4}{9}\right)+\left(\frac{1}{3}\right)\left(\frac{3}{6}\right)}$$

- 1 mark for $P(\text{box B and R})$
- 1½ marks for $P(R)$ (½ mark for each term)
- 1 mark for conditional probability fraction

$$=\frac{\frac{4}{27}}{\frac{1}{5}+\frac{4}{27}+\frac{1}{6}}$$

$$=\frac{\frac{4}{27}}{\frac{139}{270}}$$

$$=\frac{40}{139}$$

½ mark for consistent answer

4 marks

Question No. 2

Outcome: G4

Method 2

$$P(\text{box B}|R)=\frac{\left(\frac{1}{3}\right)\left(\frac{4}{9}\right)}{\left(\frac{1}{3}\right)\left(\frac{3}{5}\right)+\left(\frac{1}{3}\right)\left(\frac{4}{9}\right)+\left(\frac{1}{3}\right)\left(\frac{3}{6}\right)}$$

1 mark for $P(\text{box B and R})$

1½ marks for $P(R)$

1 mark for conditional probability fraction

$$=\frac{0.1\overline{48}}{0.5\overline{148}}$$

$$=0.288$$

½ mark for consistent answer

4 marks

Note(s):

- the calculation of $\frac{\frac{4}{9}}{\frac{3}{5}+\frac{4}{9}+\frac{3}{6}}$ in the first step is equivalent and should be given full marks
- other acceptable answers are 0.287 769… or 0.287
- give 1 mark if only a correct tree diagram is provided

EXEMPLAR

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

$$P(\text{red}) = \frac{2}{5} \times \frac{1}{3} + \frac{4}{9} \times \frac{1}{3} + \frac{3}{6} \times \frac{1}{3}$$

$$= \frac{1}{5} + \frac{4}{27} + \frac{1}{6}$$

$$= \frac{139}{270}$$

$$P(\text{red and } B) = \frac{4}{20}$$

$$= \frac{1}{5}$$

$$\frac{139}{270} \times \frac{1}{5} = \frac{139}{1350}$$

0.103 is the probability

Mark: **1½ out of 4**
Note(s):

- gave 1½ marks for P(Red)

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Question No. 3

Outcome: E2

Brandy, Leo and 6 of their friends attend a movie.

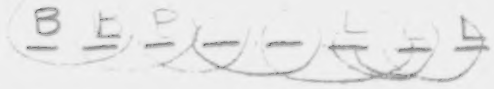
In how many ways can they be seated in a row so that Brandy and Leo do not sit next to each other?

Solution

All – together = apart

$8! - 2(7!)$	1 mark for $2(7!)$
$= 40\,320 - 10\,080$	½ mark for subtraction
$= 30\,240$ ways	½ mark for consistent answer
	2 marks

EXEMPLAR



All possible arrangements - they "do" sit together

= they "don't" sit together

$8! - 2!7 = x$

$x = 40306$ ways

Mark: **1 out of 2**

Note(s):

- deducted 1 mark for incorrect value: 2!7 should be 2(7!)

Question No. 4

Outcome: D6, D2

Solve the following equation algebraically:

$$3 \cdot 9^{x} = 5^{(2-x)}$$

Express your answer correct to 3 decimal places.

Solution

Method 1

$$3 \cdot 9^{x} = 5^{(2-x)}$$

$$3 \cdot 3^{2x} = 5^{(2-x)}$$

$$3^{2x+1} = 5^{(2-x)}$$ 1 mark for exponential theorem

$$(2x+1)\ln 3 = (2-x)\ln 5$$ 1 mark for log theorem

$$2x\ln 3 + \ln 3 = 2\ln 5 - x\ln 5$$

$$2x\ln 3 + x\ln 5 = 2\ln 5 - \ln 3$$

$$x(2\ln 3 + \ln 5) = 2\ln 5 - \ln 3$$

$$x = \frac{2\ln 5 - \ln 3}{2\ln 3 + \ln 5}$$ 1 mark for solving for x

$$= 0.557$$ 1 mark for consistent answer

4 marks

Question No. 4

Outcome: D6, D2

Method 2

$$3 \cdot 9^x = 5^{(2-x)}$$

$\log\left[3 \cdot 9^x\right] = \log 5^{(2-x)}$ — 1 mark for log theorem

$\log 3 + x \log 9 = (2-x)\log 5$ — 1 mark for log theorem

$$x \log 9 = 2\log 5 - x\log 5 - \log 3$$

$$x \log 9 + x \log 5 = 2\log 5 - \log 3$$

$$x(\log 9 + \log 5) = 2\log 5 - \log 3$$

$x = \dfrac{2\log 5 - \log 3}{\log 9 + \log 5}$ — 1 mark for isolating x

$x = 0.557$ — 1 mark for consistent answer

4 marks

Method 3

$3 \cdot 9^x = 5^2 \cdot 5^{-x}$

$9^x \cdot 5^x = \dfrac{25}{3}$

$(45)^x = \dfrac{25}{3}$

1 mark for exponential theorem

$x = \log_{45}\left(\dfrac{25}{3}\right)$ — 1 mark for log theorem

$x = \dfrac{\log\left(\dfrac{25}{3}\right)}{\log 45}$ — 1 mark for change of base formula

$x = 0.557$ — 1 mark for consistent answer

4 marks

Note(s):

- $\log 3 = 0.477\,12 \quad \log 9 = 0.954\,24 \quad \log 5 = 0.698\,97$

  $\ln 3 = 1.098\,61 \quad \ln 9 = 2.197\,22 \quad \ln 5 = 1.609\,44$
- **deduct** ½ mark if parenthesis are assumed but not written in step 1
- other acceptable answers are 0.556 987… or 0.556

EXEMPLAR

$$3.9^{x} = 5^{(2-x)}$$

$$27^{x} = 5^{2-x}$$

$$x \ln 27 = 2 - x \ln 5$$

$$x \ln 27 + x \ln 5 = 2$$

$$x(\ln 27 + \ln 5) = 2$$

$$x = \frac{2}{(\ln 27 + \ln 5)}$$

$$x = 0.408$$

Mark: **2 out of 4**

Note(s):

- deducted 1 mark for multiplying bases on line 2 (concept error)
- deducted 1 mark for not distributing on line 3

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Question No. 5

Outcome: A5

Solve the following equation:

$$\tan^2\theta + 3 = 7$$

Give the general solution in radian measure correct to 3 decimal places.

Solution

Method 1

$\tan^2\theta + 3 = 7$

$\tan^2\theta = 4$

$\tan\theta = \pm 2$

$\theta = 1.107,\ 2.034,\ 4.249,\ 5.176$

$$\theta = \begin{cases} 1.107 + 2k\pi \,|\, k \in \mathrm{I} \\ 2.034 + 2k\pi \,|\, k \in \mathrm{I} \\ 4.249 + 2k\pi \,|\, k \in \mathrm{I} \\ 5.176 + 2k\pi \,|\, k \in \mathrm{I} \end{cases}$$

1 mark for solving for $\tan\theta$
(½ mark for 2, ½ mark for -2)
2 marks
(½ mark for each solution)

1 mark for general solution

4 marks

Method 2

$\tan^2\theta + 3 = 7$

$\tan^2\theta = 4$

$\tan\theta = \pm 2$

$\theta = 1.107,\ 2.034$

$$\theta = \begin{cases} 1.107 + k\pi \,|\, k \in \mathrm{I} \\ 2.034 + k\pi \,|\, k \in \mathrm{I} \end{cases}$$

1 mark for solving for $\tan\theta$
(½ mark for 2, ½ mark for -2)
1 mark for $\theta = 1.107$
1 mark for $\theta = 2.034$

1 mark for general solution

4 marks

Method 3–Graphing Calculator

$y_1 = \tan^2 x$ OR $y_1 = \tan x$

$y_2 = 4$ $y_2 = 2$

$y_3 = -2$

1 mark for showing calculator entries

$$\theta = \begin{cases} 1.107 \\ 2.034 \end{cases}$$

$$\theta = \begin{cases} 1.107 + k\pi \,|\, k \in \mathrm{I} \\ 2.034 + k\pi \,|\, k \in \mathrm{I} \end{cases}$$

2 marks for calculating intersections
(1 mark for each value)

1 mark for general solution

4 marks

EXEMPLAR

$\tan^2\theta + 3 = 7$ (-3)

$\sqrt{\tan^2\theta} = \sqrt{4}$

$\tan\theta = \pm 2$

$\tan\theta = +2$ ($\tan^{-1}$)

$\theta_r = 63.434\,948\,82°$

QI: $63,434\ldots \cdot \frac{\pi}{180}$

$= 1.107$ rads

QIII: $243,434\,948\,88° \cdot \frac{\pi}{180}$

$= 4.249$ rads

$\tan\theta = -2$

$\theta_r = 63,434\,948\,82°$

QII: $116.565\,051\,2° \cdot \frac{\pi}{180}$

$= 2.034$ rads

QIV: $296.565\,051\,2° \cdot \frac{\pi}{180}$

$= 5.176$ rads

$\theta = 1.107, 2.034, 4.249, 5.176$

Mark: **3 out of 4**
Note(s):
- deducted 1 mark for not generalizing

Question No. 6

Outcome: E4

The 6^{th} term of the binomial expansion of $\left(x^3 - \frac{4}{x}\right)^n$ contains x^{16}.

Find the value of n.

Solution

6^{th} term ${}_nC_5 \left(x^3\right)^{n-5}\left(\frac{-4}{x}\right)^5$ — 1 mark for set up

$\frac{\left(x^3\right)^{n-5}}{x^5} = x^{16}$ — 1 mark

$\frac{x^{3n-15}}{x^5} = x^{16}$ — ½ mark for simplification

$x^{3n-20} = x^{16}$

$3n - 20 = 16$

$3n = 36$

$n = 12$ — ½ mark for consistent answer

3 marks

EXEMPLAR

$${}_nC_6 (x^3)^{n-6} \left(\frac{-4}{x}\right)^6$$

$$\frac{x^{3n-6}}{x^6} = x^{16}$$

$$3n-6-6 = 16$$

$$3n - 12 = 16$$

$$3n = 28$$

$$n = 9$$

Mark: **1½ out of 3**
Note(s):

- deducted 1 mark for incorrect set-up
- deducted ½ mark for inconsistent answer

Question No. 7

Outcome: D8

An initial population of a city was 5 000 and grew exponentially to 23 000 in 9 years. The population growth can be represented by the equation $A = Pe^{rt}$ where r is the rate of growth and t represents the number of years.

Find the rate of growth for this city correct to 3 decimal places.

Solution

Method 1

$A = Pe^{rt}$	
$23\,000 = 5\,000e^{r9}$	½ mark for correct substitution
$4.6 = e^{9r}$	½ mark for simplification
$\ln 4.6 = 9r \ln e$	1 mark for log theorem
$\frac{\ln 4.6}{9} = r$	½ mark for isolating r
$r = 0.170$	½ mark for consistent answer
	3 marks

Method 2–Graphing Calculator

$A = Pe^{rt}$	
$23\,000 = 5\,000e^{r9}$	½ mark for correct substitution
$y_1 = 23\,000$ $y_2 = 5\,000e^{x9}$	1 mark for explanation of calculator entries
Find the intersection	1 mark for explanation
$x = 0.170$	½ mark for calculating intersection
$r = 0.170$	3 marks

Note(s):

- other acceptable answers are 0.169 56 or 0.169

EXEMPLAR

$A = Pe^{Rt}$

$23{,}000 = 5{,}000e^{x9}$

$\ln 23{,}000 = \ln 5000e^{x9}$

$\ln 23000 = 9x \ln 5000$

$\frac{1}{9}\ln 5000 \quad \frac{9\ln 5000}{}$

$.131 = x$

Mark: **1½ out of 3**

Note(s):

- gave ½ mark for substitution
- gave ½ mark for isolating *r*
- gave ½ mark for consistent answer

Question No. 8

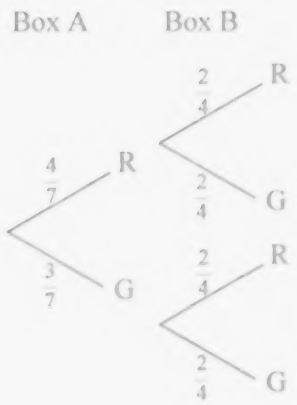
Outcome: G2

Box A contains 4 red and 3 green marbles and Box B contains 2 red and 2 green marbles. Two marbles are selected, one from each box.

Find the probability that the marbles are of different colours.

Solution

Method 1



$$P(\text{RG or GR}) = \left(\frac{4}{7}\right)\left(\frac{2}{4}\right) + \left(\frac{3}{7}\right)\left(\frac{2}{4}\right)$$ 1 mark for P (RG)
1 mark for P (GR)

$$= \frac{8}{28} + \frac{6}{28}$$ ½ mark for addition

$$= \frac{14}{28}$$

$$= \frac{1}{2}$$ ½ mark for consistent answer

3 marks

Method 2

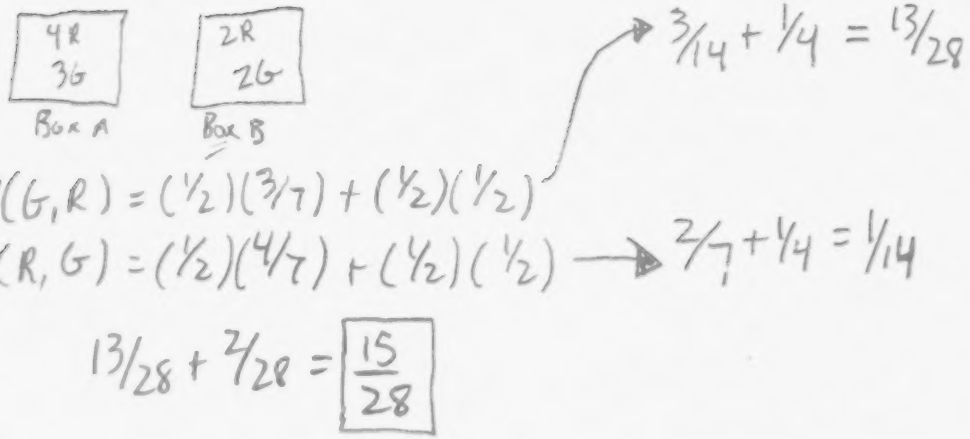
Regardless of the colour chosen from Box A, there will be a probability of ½ of choosing the other colour from Box B because Box B contains an equal number of red and green marbles. Therefore the probability that the marbles are different colours is ½.

3 marks

Note(s):

- give 1 mark if only a correct tree diagram is provided

EXEMPLAR



Mark: **1 out of 3**
Note(s):

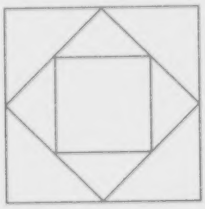
- gave ½ mark for adding P(RG) + P(GR)
- gave ½ mark for consistent answer

Question No. 9

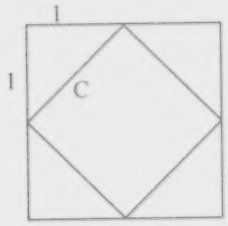
Outcome: H3

A square has a side length of 2 units. A new square is formed by joining the midpoints of each side of the original square. This process is continued infinitely.

Find the sum of the perimeters of all the squares.



Solution



$1^2+1^2=C^2$

$2=C^2$

$\sqrt{2}=C$

$\text{Perimeter}=4\,x\,C=4\left(\sqrt{2}\right)$

$t_1=8$ ½ mark for t_1

$t_2=4\sqrt{2}$ ½ mark for t_2

$r=\frac{t_2}{t_1}=\frac{4\sqrt{2}}{8}=\frac{\sqrt{2}}{2}$ 1 mark for r

$S_\infty=\frac{8}{1-\frac{\sqrt{2}}{2}}=\frac{16}{2-\sqrt{2}}$ ½ mark for substitution

½ mark for simplifying

OR

$16+8\sqrt{2}$ OR 27.313 7

3 marks

EXEMPLAR

$2 \times 4 = 8$
$\sqrt{2} \times 4 = 4\sqrt{2}$
$1 \times 4 = 4$

$r = 0.707$

$8 + 4\sqrt{2} + 4 \ldots$

$$S_\infty = \frac{t_1}{1-r}$$
$$= \frac{8}{1-0.707}$$
$$= 27.304$$

Mark: **2½ out of 3**
Note(s):
- deducted ½ mark for rounding too early

Question No. 10

Outcome: G5, G2

There are 35 students in a room; 18 of the students are girls and 17 are boys. Three students are chosen at random for a committee.

What is the probability that exactly 2 of the students are girls? Briefly explain your calculations.

Solution

Method 1

$$\frac{\overset{\text{choose girls}}{{}_{18}C_2} \cdot \overset{\text{choose boys}}{{}_{17}C_1}}{\underset{\text{choose 3 students}}{{}_{35}C_3}}$$

2 marks (½ mark for each correct combination and ½ mark for correct setup)
½ mark for explanation

$$= \frac{2601}{6545}$$

½ mark for consistent answer

$$= 0.397$$

3 marks

Method 2

Case 1: G G B

$$\frac{18}{35} \cdot \frac{17}{34} \cdot \frac{17}{33} = \frac{5\,202}{39\,270} = \frac{51}{385}$$

½ mark for explanation or identification of cases

½ mark

Case 2: G B G

$$\frac{18}{35} \cdot \frac{17}{34} \cdot \frac{17}{33} = \frac{5\,202}{39\,270} = \frac{51}{385}$$

½ mark

Case 3: B G G

$$\frac{17}{35} \cdot \frac{18}{34} \cdot \frac{17}{33} = \frac{5\,202}{39\,270} = \frac{51}{385}$$

½ mark

$$\frac{5\,202}{39\,270} + \frac{5\,202}{39\,270} + \frac{5\,202}{39\,270} = \frac{15\,606}{39\,270} = 0.397$$

1 mark for addition of cases

3 marks

Note(s)

- another acceptable answer is 0.397 40

EXEMPLAR

All restrictions / No restrictions

$\frac{{}_{18}C_2 + {}_{17}C_1}{{}_{35}C_1 + {}_{34}C_1 + {}_{33}C_1} = 1.\overline{666}$

Mark: 1 out of 3

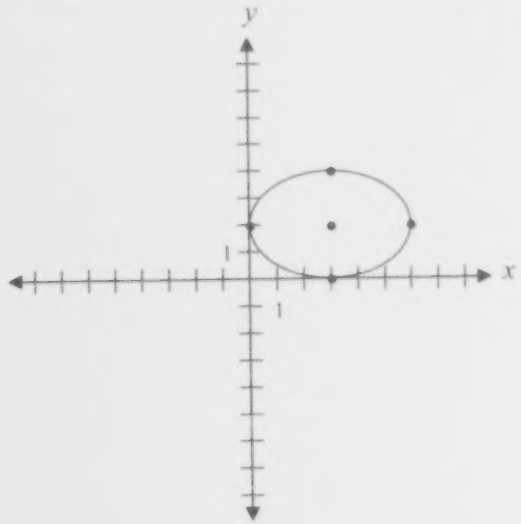
Note(s):

- gave ½ mark each for ${}_{18}C_2$ and ${}_{17}C_1$

Question No. 11

Outcome: F3

Find the equation, in standard form, of the conic section sketched below:



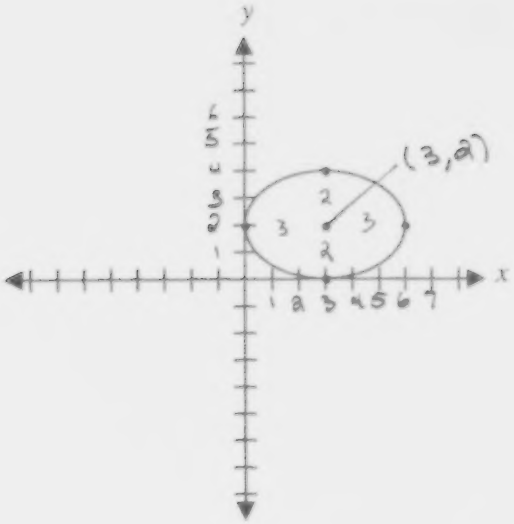
Solution

$$\frac{(x-3)^2}{9}+\frac{(y-2)^2}{4}=1$$

1 mark for centre
½ mark for the value of a^2
½ mark for the value of b^2

2 marks

EXEMPLAR



$$\frac{(x-3)}{9} + \frac{(x-2)}{4} = 1$$

Mark: **1 out of 2**

Note(s):

- gave ½ mark for the x coordinate of the centre
- gave ½ mark for the value of a^2

Part 2—Short-Answer Questions

Question No. 27

Outcome: D7, D5

Evaluate: $\ln e^2$

Answer

2 | 1 mark

Question No. 28

Outcome: A2

If the coordinates of a point $P(\theta)$ on the unit circle are $(a, -b)$, what will be the coordinates of the point $P(\theta - \pi)$?

Answer

$(-a, b)$ 1 mark

Question No. 29

Outcome: A2

Give the period of the function:

$$y = \sec\theta$$

Answer

2π | 1 mark

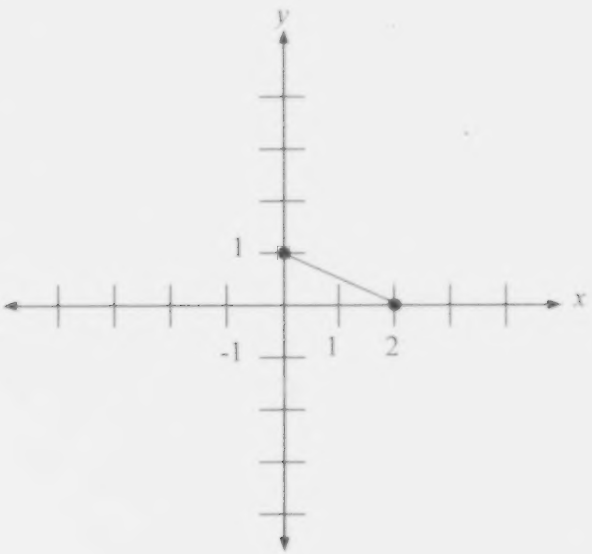
Question No. 30

Outcome: B3

Given the graph of $y = f(x)$ below:

Sketch the graph of $y = f^{-1}(x)$.

Answer

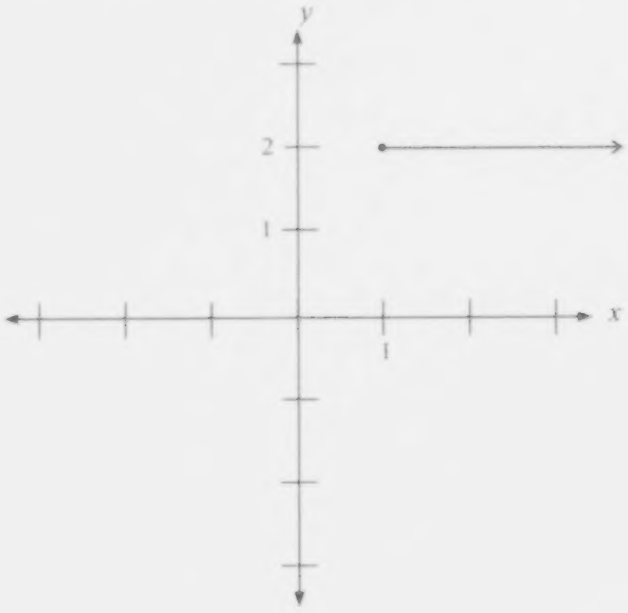


1 mark

Question No. 31

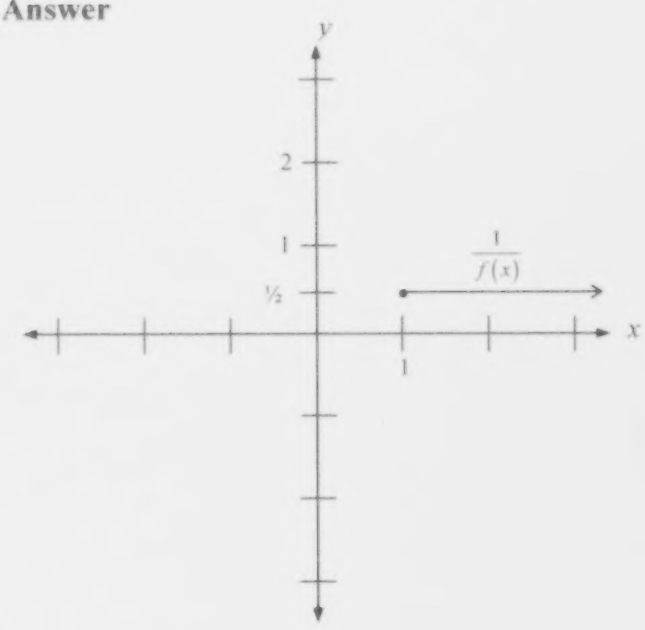
Outcome: B4

Given the graph of $y = f(x)$ below:



Sketch the graph of $y = \dfrac{1}{f(x)}$.

Answer



1 mark

Question No. 32

Outcome: C1

Express $\cot^2\theta$ in terms of $\cos\theta$ only.

Answer

$\dfrac{\cos^2\theta}{1-\cos^2\theta}$ | 1 mark

Note(s):

- give ½ mark for $\dfrac{\cos^2\theta}{\sin^2\theta}$

Question No. 33

Outcome: A1

Convert $\frac{4\pi}{5}$ radians to degree measure. Express your answer as whole number.

Answer

$\frac{4\pi}{5}\left(\frac{180}{\pi}\right)=144$

1 mark

Note(s):

- give ½ mark for simply giving $\frac{4\pi}{5}\left(\frac{180}{\pi}\right)$ OR $\frac{4(180)}{5}$

Question No. 34

Outcome: D2

Solve for x in the equation:

$$3^x = \frac{1}{27}$$

Answer

$3^x = \frac{1}{3^3}$

$3^x = 3^{-3}$

$x = -3$

1 mark

Note(s):

- give ½ mark for converting $\frac{1}{3^3}$ to 3^{-3}
- give 1 mark for $x = \frac{\ln \frac{1}{27}}{\ln 3}$

Question No. 35

Outcome: E1

Solve for n in the equation:

$$\frac{(n-1)!}{n!}=\frac{1}{5}$$

Answer

$$\frac{(n-1)!}{n!}=\frac{1}{5}$$

$$\frac{(n-1)!}{(n)(n-1)!}=\frac{1}{5}$$

$$\frac{1}{n}=\frac{1}{5}$$

$$n=5$$

1 mark

Note(s):

- give ½ mark for eliminating $(n-1)!$

Question No. 36

Outcome: E2

In how many ways can 5 people be seated at a round table?

Answer

$\frac{5!}{5}$ OR 4! OR 24 | 1 mark

Question No. 37

Outcome: E4

How many terms are in the binomial expansion of $\left(2x^2+\frac{2}{x}\right)^6$?

Answer

7 | 1 mark

Question No. 38

Outcome: F3

Give the coordinates of the centre for the conic section described by the equation:

$$\frac{(x-1)^2}{9}+\frac{y^2}{16}=1$$

Answer

(1, 0)

1 mark

Question No. 39

Outcome: F3

State the equation of an axis of symmetry for the conic section described by the equation:

$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$

Answer

$y = 0$ OR $x = 0$ | 1 mark

Question No. 40

Outcome: G2

The probability that Leigh will be on time is 0.8 and the probability that she will wear blue is 0.3. What is the probability that she will be late and wear blue?

Answer

$P(\text{B and L}) = P(\text{B}) \cdot P(\text{L})$

$= (0.3)(0.2)$

$= 0.06$

1 mark

Question No. 41

Outcome: H2

The first 3 terms of a geometric sequence are $\log 2$, $\log 2^2$, $\log 2^x$.

What is the value of x in this sequence?

Answer

$r = 2$

$\log 2^x = 2 \log 2^2$

$\log 2^x = \log 2^4$

$\therefore x = 4$

1 mark

Note(s):

- ½ mark for solving for r

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Part 2—Long-Answer Questions

Question No. 42

Outcome: C1

Prove the identity:

$$\frac{\sin x}{\cos x+1}+\frac{\cos x+1}{\sin x}=2\csc x$$

Solution

Method 1

$$\text{LHS}=\frac{\sin x}{\cos x+1}+\frac{\cos x+1}{\sin x}$$

$$=\frac{\sin^2 x+(\cos x+1)^2}{\sin x(\cos x+1)}$$ 1 mark for common denominator

$$=\frac{\sin^2 x+\cos^2 x+2\cos x+1}{\sin x(\cos x+1)}$$ ½ mark for expanding

$$=\frac{1+2\cos x+1}{\sin x(\cos x+1)}$$ 1 mark for identity

$$=\frac{2(\cos x+1)}{\sin x(\cos x+1)}$$ ½ mark for factoring

$$=\frac{2}{\sin x}$$ ½ mark for cancelling

$=2\csc x=\text{RHS}$ ½ mark for identity

4 marks

Question No. 42

Outcome: C1

Method 2

LHS

$$= \frac{\sin x}{\cos x + 1} + \frac{\cos x + 1}{\sin x}$$

$$= \frac{(\cos x - 1)}{(\cos x - 1)} \cdot \frac{\sin x}{\cos x + 1} + \frac{\cos x + 1}{\sin x}$$

1 mark for multiplying by conjugate

$$= \frac{(\cos x - 1)(\sin x)}{\cos^2 x - 1} + \frac{\cos x + 1}{\sin x}$$

$$= \frac{(\cos x - 1)(\sin x)}{-\sin^2 x} + \frac{\cos x + 1}{\sin x}$$

1 mark for identity

$$= \frac{1 - \cos x}{\sin x} + \frac{\cos x + 1}{\sin x}$$

½ mark for cancelling

$$= \frac{1 - \cos x + \cos x + 1}{\sin x}$$

1 mark for simplifying

$$= \frac{2}{\sin x}$$

½ mark for identity

$$= 2\csc x = \text{RHS}$$

4 marks

EXEMPLAR

LHS : $\dfrac{\sin x}{\cos x + 1} + \dfrac{\cos x + 1}{\sin x}$

$$\frac{\sin^2 x + (\cos x + 1)^2}{(\cos x + 1)(\sin x)}$$

$$\frac{\sin^2 x + \cos^2 x + 2\cos x + 1}{(\cos x + 1)(\sin x)}$$

$$\frac{\sin^2 x + \cos^2 x + 2}{\sin x}$$

$$2 \csc x = \text{RHS}$$

Mark: **2 out of 4**
Note(s):

- deducted 1 mark for incorrect cancellation of $\cos x + 1$
- deducted 1 mark for $\sin^2 x + \cos^2 x = 0$

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Question No. 43

Outcome: A6

a) State the domain of $y = \tan x$.

b) Sketch at least one period of the graph of $y = \tan x$.

c) Sketch the graph of $y = \tan^{-1} x$.

Solution

a)

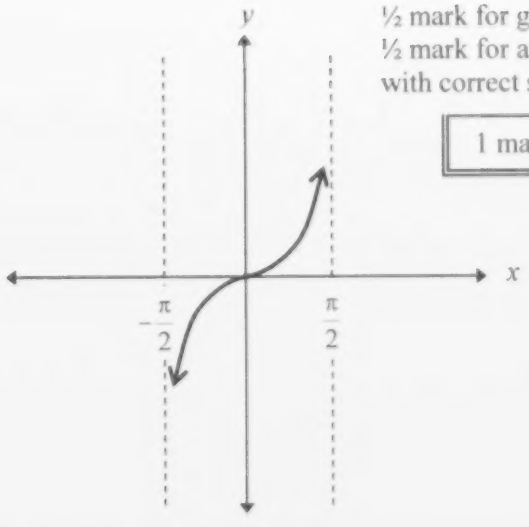
$x \neq \frac{\pi}{2} + k\pi,\ k \in \mathrm{I}$

OR

$\left\{ x \in \mathbb{R} \middle| x \neq \frac{\pi}{2} + k\pi,\ k \in \mathrm{I} \right\}$

1 mark

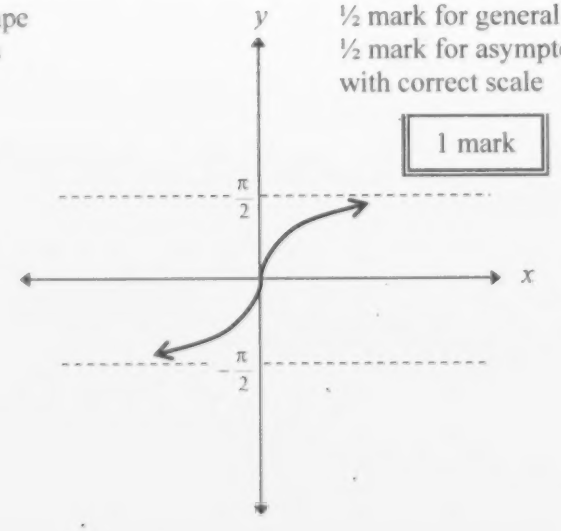
b)



½ mark for general shape
½ mark for asymptotes
with correct scale

1 mark

c)



½ mark for general shape
½ mark for asymptotes
with correct scale

1 mark

Note(s):

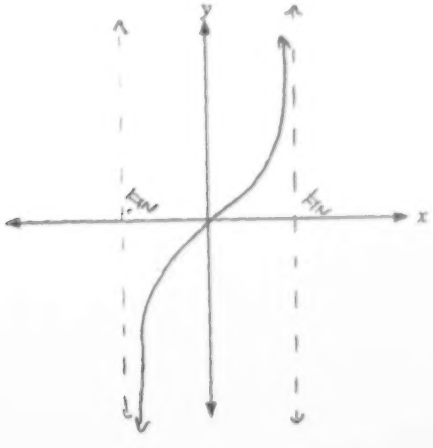
- mark part a, b, and c independent of each other
- domain can be given in degrees, $x \neq 90° + 180°k,\ k \in \mathrm{I}$
- scale on axes can be given in degrees

EXEMPLARS

a) $x \neq \frac{\pi}{2} + \pi \in I$

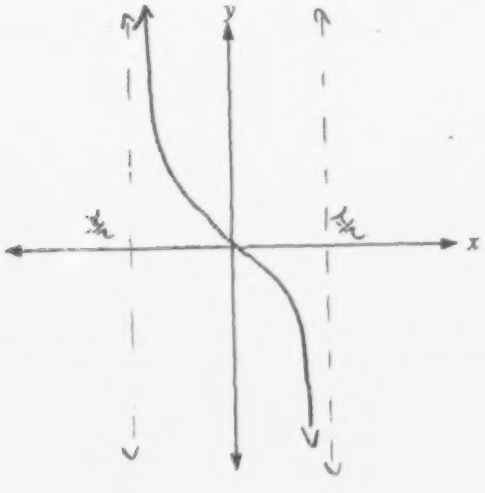
Mark: ½ out of 1
Note(s):
- deducted ½ for $\pi \in I$

b)



Mark: 1 out of 1

c)



Mark: 0 out of 1

Question No. 44

Outcome: C2

a) Find the exact value of:

$$\cos\left(\frac{11\pi}{12}\right)$$

b) Find the exact value of:

$$\sec\left(\frac{11\pi}{12}\right)$$

Solution

a)

$$\cos\left(\frac{11\pi}{12}\right)$$

$$=\cos\left(\frac{3\pi}{4}+\frac{\pi}{6}\right)$$ ½ mark for combination

$$=\cos\frac{3\pi}{4}\cos\frac{\pi}{6}-\sin\frac{3\pi}{4}\sin\frac{\pi}{6}$$

$$=\left(\frac{-\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)-\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$ 2 marks (½ mark for each value)

$$=\frac{-\sqrt{6}-\sqrt{2}}{4} \text{ OR } \frac{-\sqrt{3}-1}{2\sqrt{2}}$$ ½ mark for simplifying

3 marks

b) $\frac{4}{-\sqrt{6}-\sqrt{2}}$ OR $-\sqrt{6}+\sqrt{2}$ OR $\frac{2\sqrt{2}}{-\sqrt{3}-1}$

1 mark

Note(s):

- alternate combinations $\left(\frac{2\pi}{3}+\frac{\pi}{4}\right), \left(\frac{7\pi}{4}-\frac{5\pi}{6}\right), \left(\frac{7\pi}{6}-\frac{\pi}{4}\right), \ldots$
- mark part b) consistent with part a)

EXEMPLARS

a)

$$\frac{2\pi}{3}^{(4)} + \frac{3\pi}{4}^{(3)}$$

$$\frac{8\pi}{12} + \frac{3\pi}{12} = \frac{11\pi}{12}$$

$$\cos \cdot \cos\left(\frac{2\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \left(\sin\frac{2\pi}{3}\right)\left(\sin\frac{\pi}{4}\right)$$

$$\left(-\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$$

$$-\frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$$

$$= \frac{-1+\sqrt{3}}{2\sqrt{2}}$$

Mark: **2 out of 3**
Note(s):
- deducted 1 mark for error in formula

b)

$$\frac{-1+\sqrt{3}}{2\sqrt{2}} + 2\pi$$

Mark: **0 out of 1**

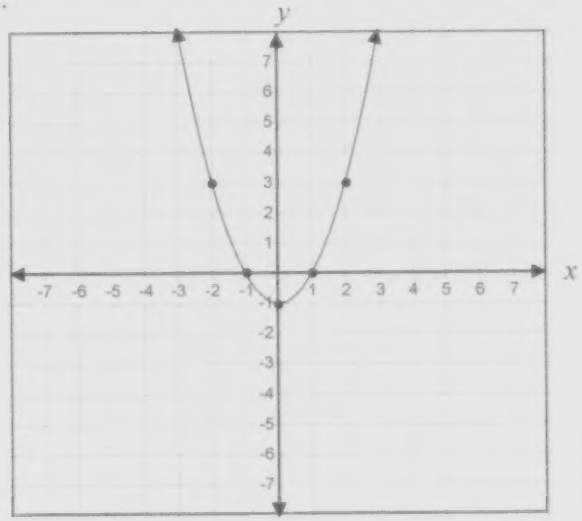
Question No. 45

Outcome: a) B1, B2 b)B5, B3

Given the graph of $y = f(x)$ below. Sketch the graphs of:

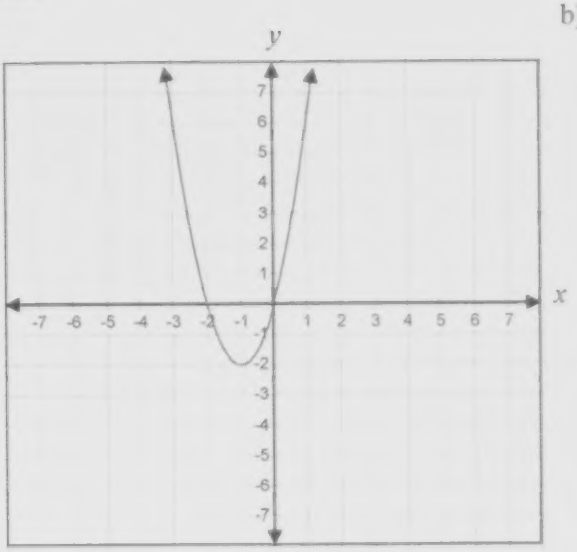
a) $y = 2f(x+1)$

b) $y = -|f(x)|$



Solution

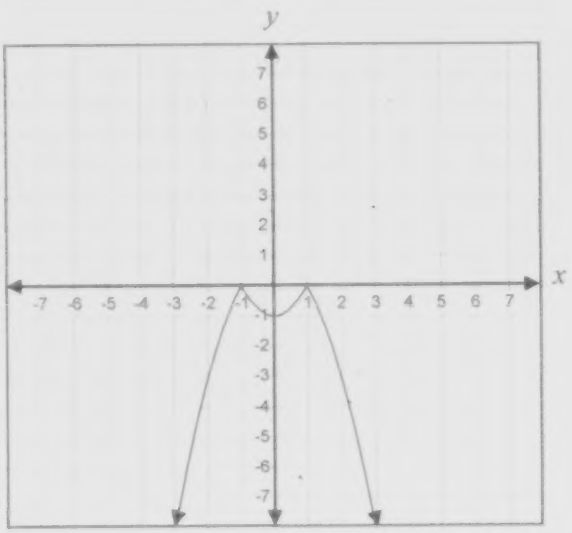
a)



1 mark for stretch
1 mark for translation

2 marks

b)



1 mark for absolute value
1 mark for reflection

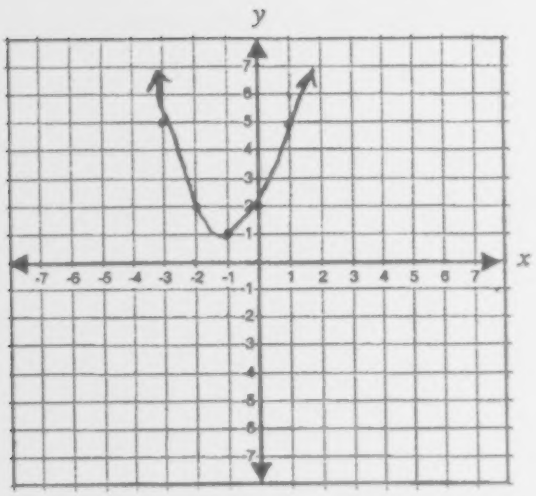
2 marks

Note(s):

- in part b) give 1 mark if student reflects and then takes the absolute value of $f(x)$

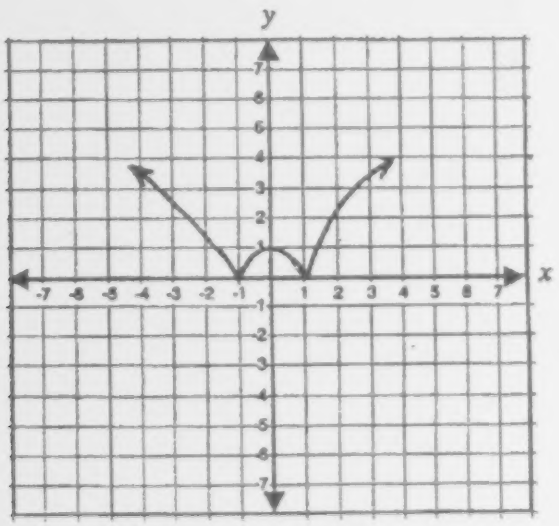
EXEMPLARS

a)



Mark: **1 out of 2**
Note(s):
- gave 1 mark for horizontal translation

b)



Mark: **½ out of 2**
Note(s):
- gave 1 mark for absolute value
- deducted ½ mark for shape

Question No. 46

Outcome: A4

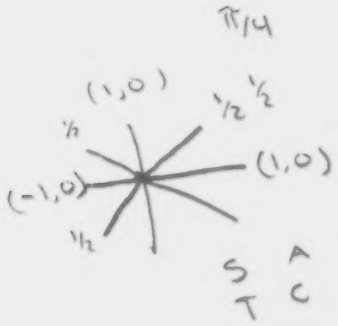
Solve for θ in the interval $[0, 2\pi]$:

$$2\cos^2\theta + 3\cos\theta + 1 = 0$$

Solution

$(2\cos\theta + 1)(\cos\theta + 1) = 0$	½ mark for factoring
$\cos\theta = -\frac{1}{2}$ or $\cos\theta = -1$	1 mark (½ mark for each value of $\cos\theta$)
$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \pi$	1½ marks (½ mark for each root)
	3 marks

EXEMPLAR



$2x^2 + 3x + 1 = 0$

$2 = 1, 2$

$3 = \frac{1}{1} + \frac{2}{2}$

$(\cos\theta + 1)(2\cos\theta + 1)$

$\cos\theta = -1 \qquad \cos\theta = -\frac{1}{2}$

$\theta = \pi \qquad \theta = \frac{2\pi}{4}, \frac{3\pi}{4}$

Mark: **1½ out of 3**

Note(s):

- gave ½ mark for factoring
- gave 1 mark for solving for $\cos\theta$
- gave ½ mark for $\theta = \pi$
- deducted ½ mark for changing the equation to an expression

Question No. 47

Outcome: F2, F3

A conic section is represented by the equation $4x^2 - 8x - y^2 + 2y - 13 = 0$.

a) Write the equation in standard form.

b) Sketch the graph of this conic section.

Solution

a) $4\left(x^2 - 2x\right) - 1\left(y^2 - 2y\right) = 13$

$4\left(x^2 - 2x + 1\right) - 1\left(y^2 - 2y + 1\right) = 13 + 4 - 1$

$4(x-1)^2 - 1(y-1)^2 = 16$

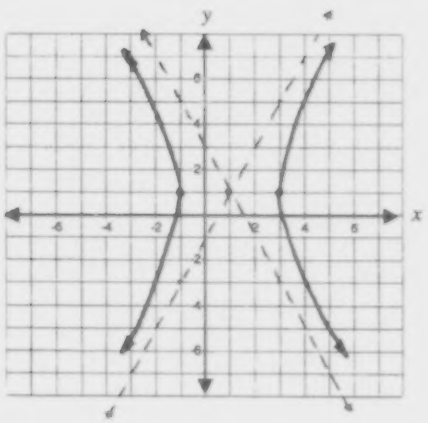
2 marks
- 1 mark for completing the square for y ($\frac{1}{2}$ mark for left side and $\frac{1}{2}$ mark for right side)
- 1 mark for completing the square for x ($\frac{1}{2}$ mark for left side and $\frac{1}{2}$ mark for right side)

$\frac{(x-1)^2}{4} - \frac{(y-1)^2}{16} = 1$

1 mark for standard form

3 marks

b)



$\frac{1}{2}$ mark for horizontal hyperbola
$\frac{1}{2}$ mark for the centre
$\frac{1}{2}$ mark for vertices consistent with the centre
$\frac{1}{2}$ mark for asymptotes

2 marks

Note(s):

- mark the graph in b) consistent with the equation given in a)

EXEMPLARS

a)

$(4x^2 - 8x\ \ ) - (y^2 + 2y - 1) = 13$

$4(x^2 - 2x + 1) - (y^2 + 2y + 1) = 13 + 4 - 1$

$\frac{4(x-1)^2}{16} - \frac{(y+1)^2}{16} = \frac{16}{16}$

$\frac{(x-1)^2}{4} - \frac{(y+1)^2}{16} = 1$

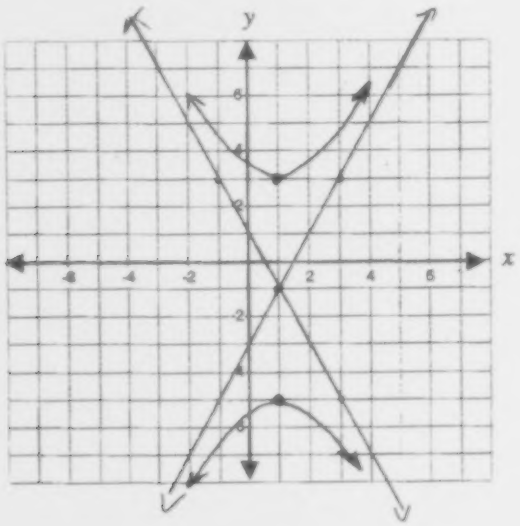
$\frac{(x-1)^2}{2^2} - \frac{(y+1)^2}{4^2} = 1$

Mark: **2 out of 3**

Note(s):

- deducted ½ mark for −1 in second brackets on line 1
- deducted ½ mark for sign error in completing the square for *y*

b)



Mark: **1 out of 2**

Note(s):

- deducted ½ mark for vertical hyperbola
- deducted ½ mark for asymptotes drawn as solid lines

Question No. 48

Outcome: D5

Express $\frac{1}{2}\log A - 3\log B$ as a single logarithm.

Solution

$\log \sqrt{A} - \log B^3$ 1 mark for log theorem

$\log\left(\frac{\sqrt{A}}{B^3}\right)$ 1 mark for log theorem

OR

$\log\left(\frac{A^{\frac{1}{2}}}{B^3}\right)$

2 marks

Note(s):

- first step need not be shown for full marks

EXEMPLAR

$$\frac{1}{2}\log A - 3\log B = \frac{\log A^{\frac{1}{2}}}{\log B^3}$$

Mark: **1 out of 2**
Note(s):

- gave 1 mark for log theorem

Question No. 49

Outcome: C2

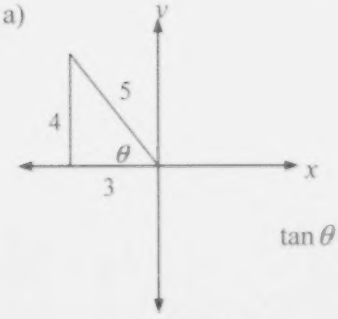
If $\sin\theta = \frac{4}{5}$ and $\sec\theta < 0$ find the exact value of:

a) $\tan 2\theta$

b) $\sin 2\theta$

Solution

a)



$\tan\theta = -\frac{4}{3}$ 1 mark for solving for $\tan\theta$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$= \frac{2\left(-\frac{4}{3}\right)}{1-\left(-\frac{4}{3}\right)^2}$$

1 mark for substitution

$$= \frac{-\frac{8}{3}}{1-\frac{16}{9}}$$

$$= \frac{-\frac{8}{3}}{-\frac{7}{9}}$$

$$= \frac{24}{7}$$

1 mark for simplifying

3 marks

b) $\cos\theta = -\frac{3}{5}$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$= 2\left(\frac{4}{5}\right)\left(-\frac{3}{5}\right)$$

½ mark for substitution

$$= -\frac{24}{25}$$

½ mark for simplification

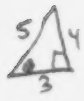
1 mark

Note(s):

- in part a) $\tan\theta = -\frac{4}{3}$ need not be stated prior to substitution into equation
- mark part b) consistent with part a)

EXEMPLARS

a)



$\tan\theta = \frac{4}{3}$

$\cos\theta = \frac{3}{5}$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(\frac{4}{3}\right)}{1-\frac{4}{3}^2} = \frac{\frac{8}{3}}{1-\frac{16}{3}} = \frac{\frac{8}{3}}{\frac{-13}{3}} = \frac{8}{-13}$$

Mark: **1½ out of 3**
Note(s):
- gave 1 mark for substitution
- gave 1 mark for simplification
- deducted ½ mark for missing brackets in denominator

b)

$$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right) = \frac{24}{25}$$

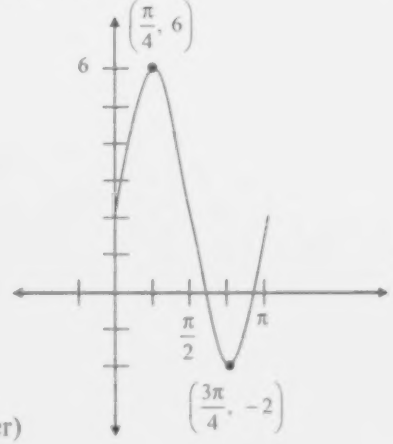
Mark: **1 out of 1**
Note(s):
- carried error from part a)

Question No. 50

Outcome: B7

The trigonometric graph shown has one maximum value at the point $\left(\frac{\pi}{4}, 6\right)$ and the next minimum value at the point $\left(\frac{3\pi}{4}, -2\right)$.

Find an equation for this graph in terms of the cosine function.



Solution

$$P = \frac{2\pi}{B} \quad \pi = \frac{2\pi}{B}$$

$A = 4$

$B = 2$

$C = \frac{\pi}{4}$

$D = 2$

2 marks
(½ mark for each parameter)

$$y = 4\cos\left[2\left(x - \frac{\pi}{4}\right)\right] + 2$$

1 mark for equation

OR

$$y = 4\cos\left(2x - \frac{\pi}{2}\right) + 2$$

3 marks

OR

$$y = -4\cos\left[2\left(x + \frac{\pi}{4}\right)\right] + 2$$

OR

$$y = -4\cos\left[2\left(x - \frac{3\pi}{4}\right)\right] + 2$$

Note(s):

- give a maximum of 2 marks for an equivalent sine function e.g., $y = 4\sin 2x + 2$

EXEMPLAR

$$y = 4\cos\left(2x - \frac{\pi}{4}\right) + 2$$

Mark: **2½ out of 3**
Note(s):
- deducted ½ mark for phase shift

Question No. 51

Outcome: B6

Given the function $y = \sqrt{x+1} - 2$:

a) the x-intercept is _____

b) the y-intercept is _____

c) Sketch a clearly labelled graph of the function.

Solution

a) $\sqrt{x+1} - 2 = 0$

$\sqrt{x+1} = 2$

$x + 1 = 4$

$x = 3$

1 mark for x-intercept

1 mark

b) $\sqrt{0+1} - 2 = y$

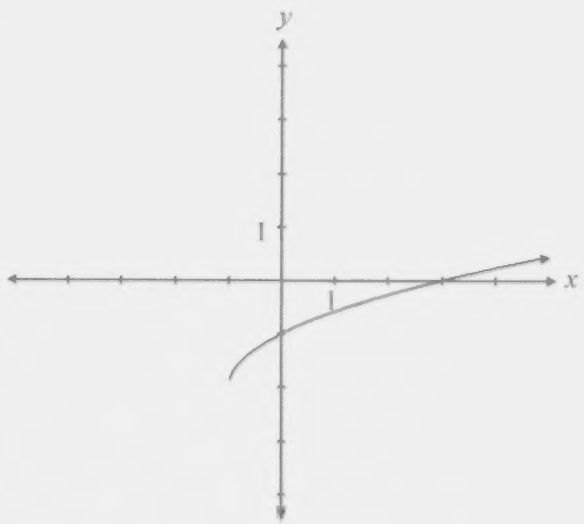
$1 - 2 = y$

$-1 = y$

1 mark for y-intercept

1 mark

c)



½ mark for general shape
½ mark for intercepts

1 mark

EXEMPLARS

a)

$$0 = (\sqrt{x+1} - 2)^2$$

$$0 = x + 1 - 4$$

$$0 = x - 3$$

$$x = 3$$

x-intercept = 3

Mark: 1 out of 1

b)

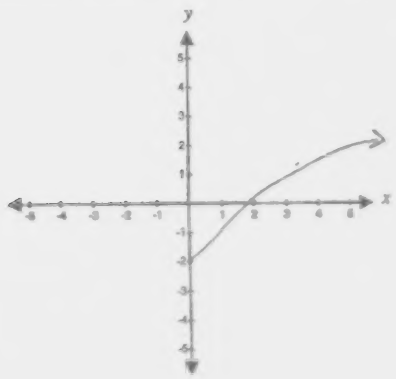
$$y = \sqrt{0+1} - 2$$

$$y = -2$$

$\therefore$ y-intercept = -2

Mark: ½ out of 1
Note(s):
- deducted ½ mark for arithmetic error

c)



Mark: ½ out of 1
Note(s):
- gave ½ mark for general shape

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Appendices

Appendix A:

MARKING GUIDELINES

- Arithmetic error, deduct ½ mark
- Concept error, deduct 1 mark
- Stating an intercept as an ordered pair, deduct ½ mark
- Changing an expression to an equation and vice versa, deduct ½ mark
- Missing or incorrect units of measure, deduct ½ mark
- Missing degree symbol in angle measure, deduct ½ mark
- Stating a correct answer in degrees instead of radians or vice versa, deduct 1 mark
- Introducing a variable without defining it, deduct ½ mark

E.g.: $2\sin x = 1$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6} \text{ over } [0, 2\pi]$$

$$x = \frac{\pi}{6} + 2k\pi \text{ or } \frac{5\pi}{6} + 2k\pi$$

Without an explanation of "k", deduct ½ mark. Student can write "$k \in \mathrm{I}$" or "k is an integer".

- Equating LHS and RHS throughout the proof of an identity, deduct 1 mark
- Omitting the symbol θ's twice or more in a trig identity or equation, deduct ½ mark
- Changing a variable without redefining it, in either an equation or an identity, deduct ½ mark for poor notation

E.g.: $2\sin\theta = 1$ $\qquad$ $\cos^2\theta = 1$

$\sin x = \frac{1}{2}$ $\qquad$ or $\qquad$ $x^2 = 1$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\qquad$ $x = \pm 1$

- Writing "$\sin x^2$" instead of "$\sin^2 x$", deduct ½ mark
- Incorrect precision or rounding, deduct ½ mark

- In a binomial theorem expansion, giving the answer as an entire term when only the numerical coefficient is requested, deduct ½ mark
- In using the log of a power theorem and omitting parentheses such as:

  a) $\log 3^{x+1} = x+1 \log 3 = x \log 3 + \log 3$, deduct ½ mark

  b) $\log 3^{x+1} = x+1 \log 3 = x + \log 3$, deduct 1 mark

- Unsimplified fractions such as: $\frac{\frac{1}{2}+\frac{1}{3}}{2}$, deduct ½ mark
- Unreduced fractions such as: $\frac{6}{8}$ or unrationalized fractions such as: $\frac{\sqrt{3}}{\sqrt{2}-1}$ are acceptable.
- If a student simplifies fractions and makes an error, deduct ½ mark.
- Teachers should be aware that in all functions, unless otherwise stated, the domain is the largest subset of the real numbers for which the function is meaningful.
- Permutations or combinations that use cases must have the cases briefly described, otherwise deduct ½ mark.
- Omitting brackets in a fraction but assuming them, deduct ½ mark

  E.g.: $\frac{2^2}{3} = \frac{4}{9}$

- Endpoints or arrowheads incorrectly shown, deduct ½ mark
- Asymptotes shown as solid lines or not shown, deduct ½ mark
- Axes not labelled or scale values not indicated (tick marks are not assumed to be one unit), deduct ½ mark

Appendix B:

IRREGULARITIES IN STANDARDS TESTS

A GUIDE FOR LOCAL MARKING

The *Administration Manual for Supervising Teachers* and the *Policies and Procedures for Standards Tests* are distributed to teachers and administrators to ensure consistency in the administration of standards tests. These documents deal with the issues of test material security and the Department's policy regarding adaptations and exemptions.

During the marking of standards tests, markers have occasionally encountered test booklets in which there have been irregularities. The following list provides examples of such irregularities, and is not intended to be exhaustive:

- Completely different penmanship in the same booklet
- Incoherent work with correct answer(s)
- Notes from a teacher indicating how he or she has assisted students during the test administration
- Student offering that he or she received teacher assistance for a question
- Student submitting work on unauthorized paper
- Evidence of plagiarism or cheating

If a student's mark on the test is 0% because he or she left the whole test booklet blank or answered with a combination of non-responses and inappropriate responses, please complete an *Irregular Test Paper Report* to confirm that the result is accurate (i.e., the student was not, in fact, absent or exempted).

Please note that student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up action at the school level. In this case, please ensure the Department is made aware that a follow-up has taken place by completing an *Irregular Test Paper Report*.

Except in the case of cheating or plagiarism where the result is a standards test mark of 0%, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an *Irregular Test Paper Report* documenting the situation, the people contacted, and the follow-up. The original copy of this report is to be retained by the local jurisdiction and a copy is to be sent to the Instruction, Curriculum and Assessment Branch, along with all other test materials.

Irregular Test Paper Report

Test: _____

Date marked: _____

Booklet ID No.: _____

Problem noted: _____

Question(s) affected: _____

Action taken or rationale for assigning marks: _____

Follow-up: _____

_____

_____

_____

_____

_____

Decision: _____

_____

_____

_____

_____

_____

_____

Marker's Signature: _____

Principal's Signature: _____

For Department Use Only—After Marking Complete

Consultant: _____

Date: _____